





# **ARITHMETIC**

Averages, Work and Rates, Ratios, and Percent Change

# **WORK AND RATES**

## Rate T

Use the picture below (rate T) to express the relationship between the parts involved in a Rate or Work question. When given more than one rate create one rate T for each separate person or job discussed.



## **Combined Efforts**

When things are working together as a team, add the rates. When things are working against each other subtract the rates.

# **Multiple Stage Trips**

When trips/jobs take place in multiple stages, calculate each stage separately then add the total times and total distance/work.

# Example:

Maria and Hector have teamed up to run different portions of a 24 mile race for a corporate charity event, with Hector running twice as far as Maria. If Hector ran at a constant speed of 8 miles per hour and Maria at a constant speed of 2 miles per hour, what was their average speed for the entire race?

First *recognize* that there are multiple rates and thus you must set up a rate T for each rate given as well as one for the overall rate. Next, fill in the numbers that were given for each of the charts and solve for the missing values.

Since Hector ran twice as far as Maria, Hector ran 16 of the 24 miles of the race and Maria ran 8 of the 24 miles of the race.

Ma	ria
8 m	iles
4 hours	2 mph
	'
niles	
4 mph	
	4 hours  Race miles



# Work and Rates | Walkthrough

- 1. If Logan walks at 10 miles per hour for 3 hours, what will be the distance he traveled?
- 4. An empty 200 gallon tank has water flowing into it from a faucet at 10 gallons per minute. If the tank has a leak that loses water at a rate of 2 gallons per minute, how long will it take for the tank to be completely full?

- 2. If Xavier paints 5 walls per hour, how many minutes will it take him to complete the 18 walls in his house?
- 5. Frank is heading from town A to town B, a distance of 33 miles, at 6 miles per hour. If Miller is heading from town B to town A at 5 miles per hour along the same route as Frank and they leave at the same time, how long will it take for Frank and Miller to meet?

- 3. Bruce can paint 18 walls in 7 hours and Wayne can complete the same job in two fewer hours. If they work together, how long will it take them to complete the job?
- 6. Peter drives from his home to his Aunt May's house at an average speed of 40 miles per hour and returns along the same route at an average speed of 60 miles per hour. Approximately what was his average speed for the entire round trip?



# Work and Rates Walkthrough | Answer Key

# **Question 1**

*Knowledge*: The rate T expresses the relationship between the components involved in rate questions simply and visually.

*Recognition*: Whenever a problem discusses a job or distance (or discusses the rate at which a job or distance is completed), draw a rate T.

*Takeaway(s)*: Rate questions typically give 2 of the 3 parts of the rate T and require you to solve for the missing part.

Answer: 30

# **Ouestion 2**

*Knowledge*: Questions involving work, rates, or distances should also be handled using rate Ts.

*Recognition*: Questions discussing tasks (work) are work and rate questions.

*Takeaway(s)*: Rate questions often involve conversion of units so be aware of the unit asked for in rate questions.

Answer: 216

# **Ouestion 3**

*Knowledge*: When people or things are working together **add the rates** in order to find the "team" rate.

Recognition: The mention of the two people working on the same job indicates that this is a "teamwork" problem. Since the people are working together there will be three Ts (one for each person and one for together).

*Takeaway(s)*: Make a new rate T for each separate event, person, or rate given.

Answer: 2 hours 55 minutes

#### **Question 4**

*Knowledge*: When people/things are working against each other **subtract the rates** in order to find the "teamwork" or combined rate.

Recognition: The mention of things working at opposite purposes (paddling upstream, flying into a headwind, one pipe filling while another empties) reveals a combined effort problem.

*Takeaway(s)*: The GMAT will require you to recognize the logic of the situation described and consider how the given situation fits within the typical GMAT problem types you have learned.

Answer: 25

# **Question 5**

*Knowledge*: When people/things are working together **add the rates** in order to find the "team" rate.

*Recognition*: When two people are working together to cover a distance it is the same as if they are working together on the same job.

*Takeaway(s)*: Problems about people meeting are typically working together problems.

Answer: 3

# **Question 6**

*Knowledge*: When there are multiple rates you should set them up in individual Ts. For multiple part jobs or multiple segment trips add the individual times and add the individual distances to find the total time and total distance.

Recognition: The mention of several parts of a trip leading to a total trip indicates a multiple segment trip. Problem Solving questions with unknowns can often be solved by replacing unknowns with numbers (upcoming lesson in Algebra).

*Takeaway(s)*: The average rate for multiple segment trips can be found without knowing the total distance.

Answer: 48 miles per hour



# Work and Rates | Approaches Applied

- 1. Two high pressure pumps, *X* and *Y*, operate independently to empty a chamber of all the gas at constant rates. If the chamber contains 30,000 liters of oxygen, how many hours will it take the two pumps to completely empty the chamber?
  - (1) Pump *X* operating alone would require 16 hours to empty the chamber.
  - (2) Pump *Y* operating alone would require 22 hours to empty the chamber.

AD | B | CE

Information Given:

Real Information:

Question Asked:

Real Question:

Translate Statements: (1)

(2)

- 2. How long did it take Mike to drive his truck nonstop from his company's distribution center to the delivery point in Tallahassee, Florida?
  - (1) Mike's average speed for the trip was 60 miles per hour.
  - (2) If Mike's average speed for the trip had been  $\frac{2}{3}$  as fast, the trip would have taken 4 hours.

AD | B | CE

Information Given:

Real Information:

Question Asked:

Real Question:

Translate Statements: (1)

(2)



- 3. Two employees work in tandem to process and route materials in their office. Worker A can process an entire shipment in 4 hours, while worker B can route an entire shipment in 6 hours. If they are working at the same time, how long will it take, in hours, for <sup>5</sup>/<sub>8</sub> of a shipment to accumulate in the office?
  - (A)  $8\frac{1}{4}$
  - (B)  $7\frac{1}{2}$
  - (C)  $6\frac{2}{3}$
  - (D)  $6\frac{1}{4}$
  - (E)  $5\frac{1}{2}$
- 4. A typist working on a document had just completed transcribing exactly half of the words when she received a particularly unsettling phone call. Before the phone call her transcription rate had been 100 words per minute, while after the call her rate dropped to just 50 words per minute. Barring the phone call, what was her average rate of transcription, in words per minute, for the entire document?
  - (A) 25
  - (B)  $33\frac{1}{3}$
  - (C)  $55\frac{1}{2}$
  - (D)  $66\frac{2}{3}$
  - (E) 75

- 1. Recognize the GMAT topic being tested.
- 2. Write down info not given.
- 3. Connect info not given to info given.
- 4. Solve.
- 5. Reread and Select.

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# Work and Rates Approaches Applied | Answer Key

#### **Question 1**

*Knowledge*: With machines or people working together the information should be put into the Rate T. More than one Rate T will be needed.

Recognition: To determine how long a job will take will require that you identify the rate(s) of the machine(s) involved. To determine the combined rate you need the rate of both machines working in concert.

*Takeaway(s)*: Rate can always be determined when the job is constant and the time required to complete the job is given.

Answer: C

## **Ouestion 2**

*Knowledge*: Determining time requires a rate and a distance. Use a Rate T to manage these elements.

Recognition: Since neither distance nor time is given, you will need to use your Rate T to determine these measurements using information from the statements.

*Takeaway(s)*: When the problem demands it, use variables in your Rate T. Very often, the variables will cancel, allowing you to answer the question.

Answer: B

#### **Question 3**

*Knowledge*: The components needed to solve a rate question can be put into the Rate T.

Recognition: The question asks how long it would take for part of a shipment to accumulate, which means that the two workers are working against one another. Because the two workers can complete the same amount of work in different times, they also work at different rates.

*Takeaway(s)*: Be aware of the different ways in which people or machines can be working against one another. Answering rate questions where two people are working together requires that you subtract their rates to find the total rate.

Answer: B

## **Question 4**

*Knowledge*: To manage different rates for different time periods use a separate Rate T for each time period.

Recognition: Since the shift in transcription rate occurred at the midway point of the job, each time period would have accomplished the same amount of work. Thus, the work in each Rate T would be a constant, and consequently you could simply choose a number for the work to help you better manage the arithmetic.

*Takeaway(s)*: Do not average the rates in a problem that includes different rates for different time periods. You must calculate total time and total work, and then find that rate.

Answer: D

