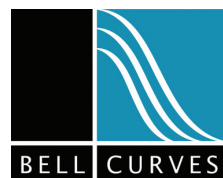


# WORKBOOK

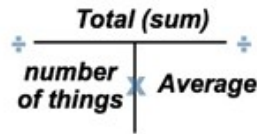


**GMAT<sup>®</sup>**  
**QUANT**

The GMAT<sup>®</sup> is a registered trademark of the Graduate Management Admission Council, which is not involved in the production of and does not endorse in this product.

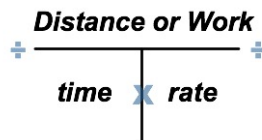
# ARITHMETIC TOOLS

## Averages



- Use the *average T* chart to calculate averages:
- Use a separate T chart for each average given or needed.
- Never *average* averages; instead, find the total sum and the total number of terms.
- If one number is below the average, then at least one number must be above the average. If one number is above the average, then at least one number must be below the average. If no number is above or below the average, then all numbers must be equal to the average.
- In any weighted average (where the number of things in each group is not the same), the overall average will be skewed toward the group with the greater number of terms.

## Work and Rates



- Use the *rate T* chart to calculate rates:
- Use a separate T chart for each thing or person.
- When things are working together, add their individual rates.
- When things are working against one another, subtract their individual rates.
- For multiple-part trips, add leg distances and leg times to get the total distance and time.
- Consider plugging in for unknowns in Problem-Solving (PS) questions.
- Be careful of unit conversions.

## Ratios and Proportions

- Ratios should always be reduced.
- For ratios with more than two parts, use a ratio grid.
- For ratios with two parts, you can use a proportion to solve or use a ratio grid.
- To combine different ratios, find a common element.
- Typically, it is best to express ratios as fractions in Data Sufficiency (DS) questions.
- Ratios are not necessarily real values.

## Percent Change

- Use the percent-change formula:  $\frac{\text{Difference}}{\text{Starting Point}}$
- Pay careful attention to the starting point.

## Arithmetic Real Information | Drill

In the following drill you will be given a statement of fact. From this statement, you should write down what you are actually being told or can determine.

**Example:** The average of three consecutive numbers is  $x$ .

This tells you that your numbers are  $x$ ,  $x + 1$ , and  $x - 1$ . This also tells you that the sum of the numbers is  $3x$ .

1. The average of 6 numbers is 142.
2. The ratio of  $x$  to  $y$  is 4 to 5.
3.  $\frac{x}{y} = \frac{4}{5}$
4. The sum of 3 numbers is 42.
5.  $a$  is 45 percent of  $b$ .
6. 12% of a group of people is male.
7. The sum of 5 distinct integers is 37.
8. At a certain company, the average experience is 11.8 years for employees with a master's degree and 6.8 years for employees with a bachelor's degree. There are more employees with master's degrees at the company.
9. The percent increase from  $a$  to  $b$  is 20.
10. John drove at a maximum speed of 50 miles per hour and at a minimum speed of 20 miles per hour for the entire 300-mile trip.

## Arithmetic Real Information Drill | Answer Key

**1. The sum of the numbers is 852.**

When faced with average information, you should use the *average T* chart to see whether you can determine additional information.

$$2. \frac{x}{y} = \frac{4}{5} \text{ or } 5x = 4y$$

Ratios can be expressed as fractions, and in Data Sufficiency questions, expressing ratios as fractions is a good way to identify the real information. When you have equations, you should also rearrange or manipulate them to find real information.

**3. The ratio of  $x$  to  $y$  is 4:5;  $x$  is 80% of  $y$**

Recognizing the common ways information is expressed and what each type of information allows you to find helps you realize here that ratios, percents, and fractions all express similar and related information.

**4. The average is 14.**

When given the total of a set of numbers, you are often being asked about averages. Use the *average T* chart to determine what additional information can be found.

$$5. a = 0.45b \text{ or } \frac{a}{b} = .45$$

Percents can also be expressed as fractions. When given percents in a word problem, you should translate them into an equation or expression and then manipulate them to find the real information.

$$6. m = 0.12p, \frac{m}{p} = .12; 88\% \text{ of the group is female, and}$$

**the total must be a multiple of 25**

Percents can often be expressed as fractions. By considering the “other group” (i.e., females when information about males is stated), you can often find information that was previously not obvious. Also, when taking a fraction of terms that are inherently integers (people, cars, etc.), be aware that the original number must be a multiple of the denominator.

**7. The average is 7.4, and all five integers are different.**

When given the total of a set of numbers, you are often being asked about averages. When averaging sets in which at least one number is different, there will be at least one number less than the average and at least one number greater than the average.

**8. The overall average is greater than 9.3.**

Since there are more people with master’s degrees, the overall average must be weighted toward those with master’s degrees.

$$9. \frac{b-a}{a} = \frac{1}{5}, \frac{b-a}{a} = \frac{1}{5}, \text{ or } \frac{b}{a} = 1\frac{1}{5}$$

Writing out and then manipulating the percent-change formula will help you find the real information here.

**10. Maximum time = 15 hours and minimum time = 6 hours**

Rate problems should be worked using the Rate T-chart. You cannot know his average speed or the exact time his trip took, but you can determine a range for the total time the trip took.

## Arithmetic Real Question | Drill

Evaluate each of the question prompts given below and determine what each question is really asking, or find another way to write the question.

**Example:** What is the average of 3 numbers?

What is the total of the three numbers?

1. If the average of 3 numbers is 142, what is the smallest number in the set?
2. If the ratio of  $x$  to  $y$  is 4 to 5, what is the value of  $x$ ?
3. What is the average of  $x$ ,  $y$ , and  $z$ ?
4. The average of the numbers in set  $n$  is 32, how many numbers are in set  $n$ ?
5. If  $a$  is 45 percent of  $b$ , what is the value of  $b$ ?
6. If Mary runs at a rate of 15 miles per hour and Jane runs at a rate of 12 miles per hour, how much longer will it take Mary to complete the Ridgewood Road Race than Jane?
7. If Mary paints at a rate of 15 walls per hour and Jane paints at a rate of 12 walls per hour, how long will it take them to completely paint Mary's house if they work together?
8. If the ratio of  $a$  to  $b$  is 3 to 7, what is the ratio of  $b$  to  $c$ ?
9. If  $x$  is the average (arithmetic mean) of three numbers, what is the value of the largest number?
10. The distance between two towns, town  $A$  and  $B$ , is 500 miles. If car  $A$  leaves town  $A$  at the same time along the same route car  $B$  leaves town  $B$ , how long will it take the two cars to meet?

## Arithmetic Real Question Drill | Answer Key

### 1. What is the sum or values of the two larger numbers?

According to the average T chart, you have enough information to determine the sum of all three numbers. If you know what the two larger values are (or the sum of the two larger values), then you can subtract that amount from the total to find the smallest number.

### 2. What is the value of $y$ ?

When given a ratio and two unknown elements, only one of the two elements is needed to determine the value of the other since the ratio gives us the relative value of the two elements.

### 3. What is the sum of $x + y + z$ ?

To find the average of a set of numbers, you need to find the total and divide by the number of elements in a set. Since you know how many numbers you have (3), you simply need the sum total of the three elements.

### 4. What is the total value of all the numbers?

To find how many numbers are in a set you need to find the total and divide by the average. Since you know the average (32), you simply need the sum of the numbers.

### 5. What is the value of $a$ ?

By translating the information given in the question into an equation, you find that you have two variables. You only need to find the value of one of the variables to solve for the other.

### 6. How long is the race?

Using the rate T-chart, you can see that the time it takes to complete the race depends on knowing the total distance. Since you know both Mary's and Jane's rates, you can calculate the time it would take each to complete the race if you know how long the race is.

### 7. How many walls does Mary's house have?

Using the rate T-chart, you can see that the time it takes to complete a job depends on the total work and the rate. Since you know both Mary's and Jane's rates, you can add their rates and calculate the time it will take them to complete the job if you know how big the job is.

### 8. What is the ratio of $a$ to $c$ ?

Since the GMAT does not give information that is not relevant in some way, the ratio of  $a$  to  $b$  must be relevant in some way. Since  $c$  is not involved in the ratio given, it must be linked in some way to  $b$ , and the only means to do that is by using  $a$  as the common element and linking the ratio given to a ratio of  $a$  to  $c$ .

### 9. What is the sum of the two smallest of the 3 numbers?

If you know the sum of the two smaller numbers, you can determine the largest number, since there are only 3 numbers.

### 10. What is the combined rate (speed) of the two cars?

Since the two cars are working together to cover the distance, you need the combined rate of the two cars to determine how long it will take to cover 500 miles.

## Work and Rates | Practice

- The men of Bravo Company are marching to capture the flag and are traveling at a rate of 2 miles every 15 minutes. The men of Alpha Company are 90 miles away and are also trying to reach the flag. Alpha Company is traveling at a rate of 3 miles every 10 minutes. If both companies start at the same time and arrive at the flag at the same time, how many miles away from the flag did Bravo Company begin?
  - 3.6
  - 40
  - 45
  - 60
  - 80
- Two men, Adam and Bill, are simultaneously constructing a brick wall. If they work together, how long will it take them to complete the wall, which is 12 feet wide, 22 feet long, and 2 feet thick?
  - Adam can complete a 3-foot-long section of the wall in 4 hours.
  - Bill can complete the entire wall in 19 hours.
- Three machines are operating simultaneously and independently at different rates. Machine *A* produces 1 widget in 4 minutes, machine *B* produces 1 widget in 3 minutes, and machine *C* produces 1 widget in 5 minutes. If two machines are to be chosen to make 20 widgets in the shortest possible time, approximately how many minutes will it take for them to produce 20 widgets?
  - 30
  - 34
  - 36
  - 44
  - 70
- On Thursday afternoon, two machines, *J* and *R*, ran continuously at uniform rates to fill a production order. At what time did they completely fill the order?
  - Both machines run at the same rate.
  - The machines had filled  $\frac{1}{2}$  of the order by 2 p.m. and  $\frac{5}{6}$  of the order by 2:20 p.m.
- When working at continuous rates, Harold can wash 12 windows in 6 hours and Melvin can wash 12 windows in  $h$  hours. Together, they can wash 18 windows in 3 hours. What is the value of  $h$ ?
  - 12
  - 9
  - 6
  - 4
  - 3
- Three hoses,  $x$ ,  $y$ , and  $z$ , each pump water at a constant rate. How long will it take the hoses to fill up a 30,000-gallon tank?
  - Hose  $x$  pumps water at 3 times the speed of hose  $z$ .
  - Hose  $y$  pumps water at half the rate of hose  $x$ .
- Two trucks are parked at Lucy's Diner. Chuck drives off in one truck at a rate of 50 miles per hour heading due east at 9 a.m. Two hours later, Patricia drives off in the other truck at a rate of 70 miles per hour heading due west. How long after Chuck's departure time will it be until they are 700 miles apart?
  - 3 hr 11 min
  - 5 hr
  - 5 hr 50 min
  - 7 hr
  - 11 hr 40 min
- Cyclist *A* leaves point *X* at noon and travels at a constant velocity in a straight path. Cyclist *B* leaves point *X* at 2 p.m. and travels along the same path at a constant velocity. Cyclist *B* overtakes cyclist *A* at 4 p.m. At what speed was cyclist *B* traveling?
  - Cyclist *A* traveled 15 miles in the first hour.
  - The rate of cyclist *B* is twice that of cyclist *A*.

9. Three machines,  $A$ ,  $B$ , and  $C$ , can complete a certain task in 10 hours, 4 hours, and 5 hours respectively. If all 3 machines worked together on that task for 1 hour and then stopped, how many hours would it take machine  $C$  to complete the task?
- (A)  $\frac{4}{5}$   
 (B)  $2\frac{1}{4}$   
 (C)  $2\frac{3}{4}$   
 (D) 4  
 (E)  $6\frac{1}{2}$
10. George and Barbara are each soliciting donations for a different charity and will each visit all houses in community  $X$ . They will visit houses together for an hour and then afterward will visit houses separately. Does Barbara collect more money than George does?
- (1) When visiting the same house, George always receives a donation twice as large as the donation received by Barbara.  
 (2) Barbara averages \$2 less per house than George.
11. It takes Kaya a total of 9 hours to go from home to work and back again on her bike. She travels at an average speed of 5 miles per hour from home to work and twice as fast coming home. If Kaya travels by the same route in both directions, then what is the distance in miles from her home to work?
- (A) 67.5  
 (B) 45  
 (C) 30  
 (D) 18  
 (E) 6
12. Painters  $X$  and  $Y$  are simultaneously painting a house. They can paint the house together in 25 hours. How many more hours would it take painter  $X$  to paint the house alone than working together with painter  $Y$ ?
- (1) Painter  $X$ 's rate is  $\frac{4}{5}$  that of painter  $Y$ 's.  
 (2) Painter  $Y$  could paint the house alone in 61 hours.
13. The rearmost end of a 66-foot-long truck exited a 330-foot tunnel exactly 6 seconds after the foremost end of the truck entered the tunnel. If the truck traveled the entire tunnel at a uniform speed, what was the speed of the truck in miles per hour (1 mile = 5,280 feet)?
- (A) 225  
 (B) 90  
 (C) 45  
 (D) 37.5  
 (E) 27
14. Jason must drive from his home to his office by one of two possible routes. If he must also return by one of these routes, what is Jason's average speed on a roundtrip from his home to his office?
- (1) The longer route is 3 times the length of the shorter route.  
 (2) Jason's average speed while driving to his office along the longer route was 50 miles per hour, and his average speed while returning home along the shorter route was 60 miles per hour.
15. Ramon and Jason laid bricks for the six walls of a new building together, each of them working alone at a constant rate. Ramon spent twelve fewer hours laying bricks than Jason did, but Ramon laid bricks at a rate fifty percent faster than Jason. If each wall required 204 bricks and it took the two men a total of 84 hours to complete the job, how many bricks per hour did Ramon lay?
- (A) 24  
 (B) 22  
 (C) 18  
 (D) 14  
 (E) 12



## Work and Rates Practice | Answer Key

Answer .....Online Question ID

1. **B** ..... (322)
2. **C** ..... (84)
3. **B** ..... (327)
4. **B** ..... (7779)
5. **E** ..... (324)
6. **E** ..... (7786)
7. **D** ..... (321)
8. **A** ..... (7796)
9. **B** ..... (333)
10. **B** ..... (2257)
11. **C** ..... (330)
12. **D** ..... (75)
13. **C** ..... (336)
14. **C** ..... (6726)
15. **C** ..... (9128)